Indirect Optimal Control for Minimum-Fuel Low-Thrust Earth-Moon Transfer

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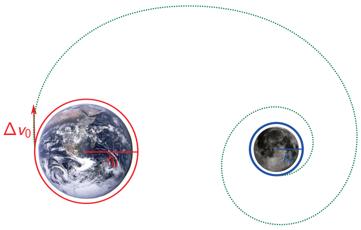
Introduction

Objective

Determine low-thrust minimum-fuel transfers between a Low Earth Orbit (LEO) and a Lunar Orbit (LO) using Low-Thrust propulsion.



Dynamics of the problem

The dynamics is based on the Circular Restricted Three-Body Problem (CR3BP) to which is added the Sun's perturbation leading to the Bicircular Restricted Four-Body Problem (BR4BP) given by:

Equations of motion

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{v}_x = 2v_y + \Omega_{s,x} + \dfrac{u_1 F_{max}}{m} \\ \dot{v}_y = -2v_x + \Omega_{s,y} + \dfrac{u_2 F_{max}}{m} \\ \dot{m} = -\|\vec{u}\| \dfrac{F_{max}}{I_{sp} g_0} \end{cases}$$

State: $\vec{\xi} = (x, y, v_x, v_y, m)^T$
Control: $\vec{u} = (u_1, u_2)^T$
$\dot{\vec{\xi}} = \varphi(t, \vec{\xi}, \vec{u})$

Where:

$$\Omega_{s,x} = \frac{\partial \Omega_s}{\partial x}, \quad \Omega_{s,y} = \frac{\partial \Omega_s}{\partial y}$$

$$\Omega_s = \Omega + \frac{m_s}{r_s} - \frac{m_s}{\rho_s^2}(x\cos\omega + y\sin\omega)$$

$$\Omega = \frac{x^2+y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2}$$

$$\omega = \omega_s t + \omega_0$$

$$r_1^2 = (x+\mu)^2 + y^2$$

$$r_2^2 = (x+\mu-1)^2 + y^2$$

$$r_s^2 = (x - \rho_s\cos\omega)^2 + (y - \rho_s\sin\omega)^2$$

Optimal Control formulation

The problem can be formulated as follows

$$(P) \begin{cases} \text{Find } \{\vec{u}, t_f\} = \underset{\vec{u}, t_f}{\text{argmin}} \; -Km(t_f), \quad K > 0 \\ \dot{\vec{\xi}} = \varphi(t, \vec{\xi}, \vec{u}) \\ \|\vec{u}\| \le 1 \\ h_0(\vec{\xi}(t_0)) = 0 \\ h_f(\vec{\xi}(t_f)) = 0 \end{cases}$$

Initial conditions: circular Earth orbit of radius $r_0 = R_e + H_0$, tangential velocity increment with modulus $\Delta v_0 \le 3500$ m/s.

Final conditions: circular Lunar orbit of radius $r_f = R_m + H_f$.

Solution method: Monte Carlo-based shooting algorithm

Pontryagin's Maximum Principle

- Hamiltonian: $H(t, \boldsymbol{\xi}, \boldsymbol{p}, \boldsymbol{u}) = \boldsymbol{p}^T \varphi(t, \boldsymbol{\xi}, \boldsymbol{u})$
- Optimal control:

$$\boldsymbol{u} = -\eta \frac{\vec{p}_v}{\|\vec{p}_v\|}, \qquad \eta = \begin{cases} 0 & \text{if } \mathcal{S}(m, \boldsymbol{p}) > 0 \\ \eta \in [0,1] & \text{if } \mathcal{S}(m, \boldsymbol{p}) = 0 \\ 1 & \text{if } \mathcal{S}(m, \boldsymbol{p}) < 0 \end{cases}$$

The optimal control is bang-off-bang!

where $\vec{p}_v = (p_{v_x}, p_{v_y})^T$ and $\mathcal{S}(m, \boldsymbol{p}) = -p_m - \dfrac{I_{sp} g_0}{m}\|\vec{p}_v\|$

- Costate dynamics: $\dot{\boldsymbol{p}} = -\nabla_{\boldsymbol{\xi}} H(t, \boldsymbol{\xi}, \boldsymbol{p}, \boldsymbol{u})$
- Transversality conditions: $TC_0(\boldsymbol{\xi}(t_0), \boldsymbol{p}(t_0)) = 0$, $TC_f(\boldsymbol{\xi}(t_f), \boldsymbol{p}(t_f)) = 0$
- Free terminal time: $H(t_f, \boldsymbol{\xi}(t_f), \boldsymbol{p}(t_f), \boldsymbol{u}(t_f)) = 0$

Massive exploration of the space of variables

- Shooting function: backward propagation of states and costates
- Dynamics sensitivity close to the Moon is circumvented
- "Monte Carlo" sampling of the unknown variables to find the zeros of the shooting function → no use of Newton's method
- Smoothing the control is not necessary

Algorithmic tricks to reduce computational burden

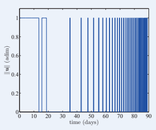
Thrust vector $u(t_f)$ in **opposite** direction of **velocity** vector
A **bisection algorithm** is used to obtain the initial Earth orbit
The value of Δv_0 can be easily computed a posteriori

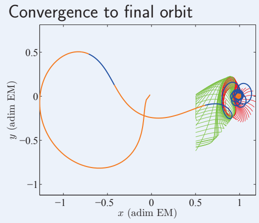
Results

Family A

Family A is made of medium-duration transfers with an arrival to the Moon region through the L_1^{EM} neck.
Three phases have been observed

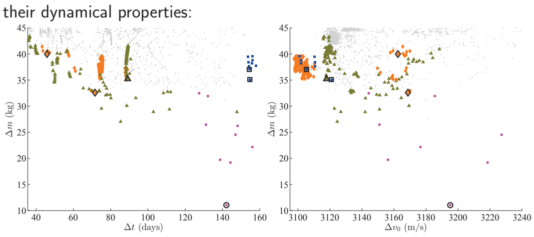
1. L_1^{EM} stable invariant manifold
2. L_1^{EM} unstable invariant manifold
3. Convergence to final orbit





Left: trajectory of a sample transfer of family A. **Top:** Thrust profile of the same sample transfer.

More than 17000 transfer trajectories have been found by the procedure. Each one is the solution of a particular instance of the problem associated with specific values of ω_0 and Δv_0.
The successful transfers have been classified in 4 main families according to their dynamical properties:

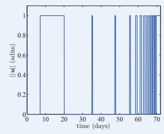


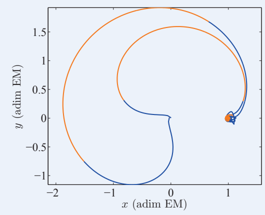
Families **A** and **B**: short to medium time of flight Δt
Families **C** and **D**: long time of flight Δt
Family **C**: smallest fuel consumption Δm

Family B

Family B consists of medium-duration transfers arriving to the Moon region through the L_2^{EM} neck.
Three phases are observed:

1. L_2^{EM} exterior stable manifold
2. L_2^{EM} interior unstable manifold
3. Convergence to final orbit





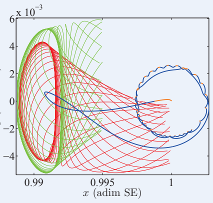
Left: trajectory of two sample transfers of family B. **Top:** Thrust profile of one of the sample transfers.

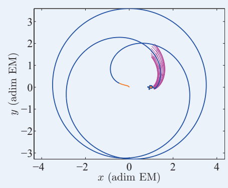
Family C

Family C consists in long-duration transfers using the Sun-Earth manifolds in addition to the Earth-Moon dynamical structure.
Three phases are distinguished in the transfer:

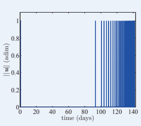
1. Sun-Earth phase
   - L_1^{SE} stable manifold
   - L_1^{SE} unstable manifold
2. Earth-Moon phase
   - L_2^{EM} stable manifold
   - L_2^{EM} unstable manifold
3. Conv. to final orbit

Observe that in the first two phases the thrust is off and is turned on for the last phase around the Moon. This is why this family leads to the lowest fuel consumption.





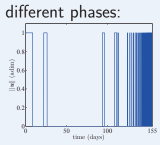
Left: trajectory of a sample transfer of family C in the Sun-Earth rotating frame. **Center:** trajectory of the same transfer in the Earth-Moon rotating frame. **Right:** Thrust profile of the same transfer.

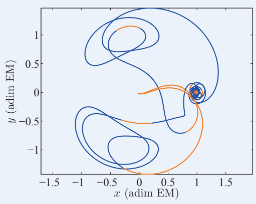


Family D

Family D consists in transfers of long duration arriving to the Moon through L_4^{EM}/L_5^{EM} regions. They are characterized by three different phases:

1. To L_4^{EM}/L_5^{EM} practical stability regions
2. From L_4^{EM}/L_5^{EM} to Moon's region
3. Convergence to final orbit via L_2^{EM}/L_1^{EM}





Left: trajectory of two sample transfers of family D. **Top:** Thrust profile of one of the transfers.

Conclusions

- In this study indirect optimal control theory has been applied for the first time to the PBR4BP.
- The Monte Carlo-based shooting method has proved to be efficient:
  - Numerical issues of standard shooting methods are avoided
  - A huge number of locally optimal trajectories can be generated
- The transfer trajectories have been classified according to the dynamic structures of the underlying CR3BPs

Future Work

- Computation of minimum-fuel transfers in the three-dimensional BR4BP
- Application of the Monte Carlo shooting method to other problems

Contact

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