Analytical and statistical characterization of the long term behavior of a cloud of debris generated by a break-up in orbit.

Université de Lille 1 SCIENCES ET TECHNOLOGIES

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Abstract

This paper provides a statistical study of the time required to form a cloud that can be considered as a randomly distributed one around the Earth after a break-up of a satellite. Starting with a simplified break-up model, we characterize, thanks to a statistical approach, if and when the distribution of the space debris within the cloud can be considered as random. Differences in the values of mean changes of velocity within the cloud that enable to describe the variations induced on the orbital keplerian elements are studied. The sensitivity of the approach is investigated, and a comparison with Fengyun-IC TLE data sets is provided.

1. Effects of ΔV on the values of keplerian orbital elements.

A ΔV vector can be expressed in the local frame defined by the velocity of the parent body, with three components classically defined in the along-track, across-track and out of planes components: $\Delta V_T, \Delta V_N, \Delta V_W$.

- Semi-major axis:

We have :

$$\Delta a = \frac{2a^2}{\mu} V.\Delta V_T$$

- Eccentricity:

Knowing that:

$$e = \sqrt{1 + \frac{2hC^2}{\mu^2}}$$

and based on the dependency of the areal velocity C and the energy constant h with the position and velocity of the satellite, we get:

$$\Delta e \approx \frac{x_N^2 V}{\mu^2 e}\left(V^2 - \frac{\mu}{a}\right)\Delta V$$

- Inclination:

$$\Delta i \approx \frac{1}{C_0 \sin(i_0)}(\Delta C \cos(i) - \Delta C e_z)$$

The way these expressions are obtained is provided in the paper, which contains in particular further information about the approximations. These expressions can be used to derive the nodal and apsidal closure times, from the Δ_V considered here as an input, and that can be inserted into the secular rates of the right ascension of the ascending node Ω and the change of mean motion Δn due to J_2:

$$\dot{\Omega}_{J_2} = -\frac{3}{2} n J_2 \left(\frac{R_0}{a}\right)^2 \frac{1}{(1-e^2)^2}\cos(i) = f_\Omega(a, e, i)$$

$$\Delta n_{J_2} = \frac{3}{2} n J_2 \left(\frac{R_0}{a}\right)^2 \frac{1}{(1-e^2)^{\frac{3}{2}}}(3\cos^2(i) - 1) = f_{\Delta n}(a, e, i)$$

The analytical conclusions are based on the fact that:

$$\Delta\dot{\Omega} = \frac{\partial f_\Omega}{\partial a}\Delta a + \frac{\partial f_\Omega}{\partial e}\Delta e + \frac{\partial f_\Omega}{\partial i}\Delta i$$

$$\Delta(\Delta n) = \frac{\partial f_{\Delta n}}{\partial a}\Delta a + \frac{\partial f_{\Delta n}}{\partial e}\Delta e + \frac{\partial f_{\Delta n}}{\partial i}\Delta i$$

2. Statistical approach: point processes

Let *X* be a point process, namely a mathematical model describing random configurations of points. These configurations are observed in an observation window denoted *W* and with an intensity λ. The intensity of a point process corresponds to the mean number of points per volum unit.

For this study, we make the following hypotheses: (i) the population of space debris is a realization of a point process (i.e.: a configuration of points), (ii) the point process is considered to be locally stationary. The second hypothesis implies that a translation of the point configuration will not change its statistical properties.

The results will not depend on how it is decided to define the observation window and λ will be the same everywhere in *W*:

$$\lambda = \text{cst} = \frac{\mathbb{E}\text{Number of points}}{\text{Volume of W}} = \frac{\mathbb{E}N(X)}{\nu(W)}. \quad (1)$$

We use the "nearest-neighbour distance distribution function" *G(r)*[1], defined as follows. Let *b(x,r)* be a ball or radius $r > 0$ centred in *x* a point of the point process *X* and again, let $\rho(x, X)$ be the minimum Euclidean distance between *x* and another point of *X*. The *G*-function describes the probability of finding a point of *X* in that ball without counting *x* itself,

$$G(r) = \mathbb{P}_x(\rho(x, X\backslash\{x\}) \leqslant r). \quad (2)$$

The way the G(r) function is computed in practice is of great importance. The problem linked to the estimation part is that there might exist interactions between points that lie outside the observation window. Only the points which are located at a minimal distance from the border are taken under consideration. We have :

$$\hat{G}(r) = \frac{\sum_{x\in X} \mathbb{1}\{\rho(x, X\backslash\{x\}) \leqslant r\}.\mathbb{1}\{\rho(x, \partial W) > r\}}{\sum_{x\in X} \mathbb{1}\{\rho(x, \partial W) > r\}}. \quad (3)$$

The results can then be compared with those obtained for a point process with known analytical formulas for the function *G(r)*. We chose the Poisson point process in this study because it is considered as the reference model when there is no interaction between the points in a sample. Deviations from the value of the summary characteristic obtained for the Poisson process give a hint on the behaviour of the point process. Equation 4 gives the G-function for the Poisson process for this study (in 3 dimensions) [1, 6]. Letting λ be the intensity of the point process and *d* the number of dimensions considered in the study ($d = 3$),

$$G_{Poisson}(r) = 1 - \exp(-\lambda\frac{4}{3}\pi r^3) \quad (4)$$

The G function enables as well to define a probability of collision within a cloud of objects.

3. An example: the Fengyun-IC test case

The destruction of the Fengyun IC satellite, on 11 January 2007, was the biggest disaster in orbit with the creation of more than 3,000 fragments with a size upper than 10 cm, identified and included in the TLE catalog of the USSTRATCOM [7]. It occured at an altitude of 863 km and the cloud spread until an altitude of 4000 km. Thus, projections showed that only the half will reentry after 2025 [8]. Moreover, the studies of the orbital environment showed that, in almost every simulation of the space debris environment for the next century, at least one collision imply a fragment of Fengyun IC.

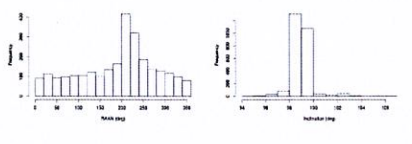

Figure 1: *Distribution of the inclinations and right ascensions of the ascending nodes of the cloud of debris from the Fengyun-IC event, July 2017, from celestrak.com*

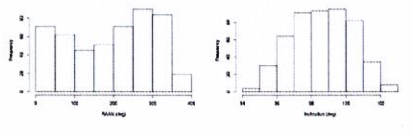

Figure 2: *Distribution of the inclinations and right ascensions of the ascending nodes of the cloud of debris simulated with the STELA software*

Chi-Square hypothesis tests of normality and uniformity for the distributions in inclination and right ascension of ascending node respectively have been conducted for the two previous cases. From the TLE data set:

Variable	χ^2_{obs}	χ^2_{theo}	Conclusion
RAAN	888.304	24.996	Rejected
Inclination	356.381	18.307	Rejected

Results of the Chi-Square test of uniformity/normality on the distributions of RAAN and inclination from the TLE data set

From the STELA propagation:

Variable	χ^2_{obs}	χ^2_{theo}	Conclusion
RAAN	59.016	11.07	Rejected
Inclination	17.604	12.592	Rejected

Results of the Chi-Square test of uniformity/normality on the distributions of RAAN and inclination from the STELA propagation

From the computation with the STELA software, different data sets are available for several instants. Figure 3 shows the evolution with time of the probability computed with the G-function and for different radii, namely 100 (blue), 250 (red) and 400 km (light-blue).

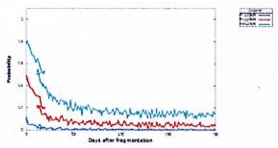

Figure 3: *Evolution with time of the probability given by the G-function for a radius equal to 100, 250 and 400 km*

4. Transport Equation / Diffusion Equation

By using and solving these types of equations (**Transport**: Equation (6), **Diffusion**: Equation (5)), the evolution over space and time of a cloud of space debris can be modelled as the one of a fluid (containing a given number of particules N_V).

$$\frac{\partial N_V}{\partial t} - D\frac{\partial^2 N_V}{\partial x^2} = 0 \quad (5)$$

$$\frac{\partial N_V}{\partial t} + V\frac{\partial N_V}{\partial x} = 0 \quad (6)$$

5. Conclusions and prospects

Statistical tests applied to a synthetic population or to a TLE data set can lead to very different results in terms of acceptability of randomness. The example of the Fengyun-IC case shows that the expression of apsidal and nodal closure times, even if of great importance to characterize the geometry of a cloud and its main evolution with time, are parameters that have to be complemented by other parameters – or statistical laws – to quantify the probability of collisions inside the cloud. A next step of this study will account for the effects of the atmospheric drag on the results presented here, but also for other kinds of events like collisions in orbit. Another one will be to add the direction of the velocity vector in the computation of the G-function (or equivalent) as a parameter or *mark*. Finally, the resolution of the equations presented in Section 4 would allow a comparison between the two propagation methods, and another possibility to assess the risk for an orbiting system to be hit by a fragment generated by an event in orbit (explosion/collision).

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